Studying the decay of the vacuum energy with the observed density fluctuation spectrum

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(Received 21 May 2004; published 22 September 2004)

We investigate here models that suggest that the vacuum energy decays into cold dark matter (CDM) and show that the density fluctuation spectrum obtained from the cosmic microwave background (CMB) data together with large galaxy surveys (e.g., the Sloan Digital Sky Survey), puts strong limits on the rate of decay of the vacuum energy. CDM produced by a decaying vacuum energy would dilute the density fluctuation spectrum, created in the primordial universe and observed with large galaxy surveys at low redshifts. Our results indicate that the decay rate of the vacuum energy into CDM is extremely small.

DOI: 10.1103/PhysRevD.70.063529 PACS numbers: 98.80.–k, 04.62.+v, 95.35.+d, 98.70.Vc

I. INTRODUCTION

Bronstein (1933) was the first to introduce the idea that the vacuum energy could decay by the emission of matter or radiation [1]. Later, a wide variety of phenomenological models for the decay of vacuum energy were suggested (e.g., [2–13]). A particularly interesting model was that of Freese *et al.* [6], who assumed that the vacuum energy density ρ_v is related to the relativistic matter density ρ_r as $x \equiv \rho_v/(\rho_r + \rho_v) =$ constant, where $\rho_r = \rho_e + \rho_\gamma + \rho_\nu$ ($e^+ e^-$ pairs, photons, and N_ν species of neutrinos). They noted that x must be less than 0.07 in order to have produced the observed ratio of baryons to photons in the Universe in the nucleosynthesis epoch. In their model, the vacuum energy density, $\rho_v = [x/(1-x)]\rho_r$, decreases at a similar rate as does ρ_r since x is constant.

Birkel and Sarkar [7] also studied the model of Freese *et al.* However, they assumed that the vacuum energy only decays into photons: $x \equiv \rho_v/(\rho_\gamma + \rho_v)$. They also assumed that x was constant during the evolution of the Universe. From the condition that the decay of the vacuum energy density must be consistent with primordial nucleosynthesis abundances, they found an upper x limit, $x_{\max} = 0.13$, which corresponds to $\rho_v < 4.5 \times 10^{-12}$ GeV^4 (for a nucleon-to-photon ratio $\eta \simeq 3.7 \times 10^{-10}$). This value for ρ_v is orders of magnitude greater than the present value obtained from recent Type Ia supernovae data, $\Lambda_0 = \Omega_\Lambda^0 \rho_c^0 \simeq 6h_0^2 \times 10^{-47}$ GeV^4 [14,15]. Since $\rho_\gamma \ll \rho_v$ at present, $x \sim 1$, which is much greater than the value $x_{\max} = 0.13$, found by Birkel and Sarker and $x_{\max} = 0.07$, found by Freese *et al.* Thus, these constant x models are inconsistent with observational data.

Nonsingular deflationary cosmology models, considered by Lima and Trodden [8], were also discussed by Birkel and Sarkar [7]. These models are a generalization of the model of Freese *et al.* with x given by $x' \equiv \rho_v/(\rho_v + \rho_m + \rho_r) = \beta + (1-\beta)(H/H_I)$, where β is a dimensionless constant of order unity, ρ_m is the nonrelativistic matter density, and H_I is the inflationary Hubble parameter. In this generalized model, x' is not strictly a constant since it varies with H. It only becomes constant when $H \ll H_I$ (i.e., for times much greater than the inflation era). Lima and Trodden [8] required that $\beta \geq 0.21$ due to the age of the Universe. However, Birkel and Sarkar argued that these nonsingular deflationary cosmological models are invalid since $\beta < 0.13$ from primordial nucleosynthesis data.

Overduin *et al.* [9] studied the x parameter of Freese *et al.* using a step function, $x(t) = x_r$ when $t < t_{\rm eq}$, the equipartition time when the matter density is equal to the radiation density, and $x(t) = x_m$ when $t > t_{\rm eq}$. They found that x can not exceed 0.001, in order not to distort the CMB spectrum. Since, at present ($z \sim 0$), x is close to unity, this value is inconsistent with a constant x.

In this article, we do not assume a constant x as do Birkel and Sarker, Overduin *et al.*, and Lima and Trodden (for times much greater than the inflation era). We assume only that the vacuum energy decays into CDM as a function of the redshift between the recombination era and the present. If the vacuum energy decays into CDM, increasing ρ, the $\delta\rho/\rho$ spectrum observed at low redshifts would have been diluted and the $\delta\rho/\rho$ would have been bigger at the recombination era. We examine to what extent the vacuum energy density can vary with redshift from the recombination era ($z \sim 1070$) to the present ($z \sim 0$), based on recent data of the CMB anisotropies.

The density fluctuations obtained by the 2dF galaxy redshift survey (2dFGRS) were compared with the measurements of the CMB anisotropies by Peacock *et al.* [16]. They analyzed the average value of the ratio of the galaxy to the matter power spectra, defining a bias parameter, $b^2 \equiv P_{gg}(k)/P_{mm}(k)$, over the range of wave numbers $0.02 < k < 0.15h$Mpc^{-1}. The scale-independent bias parameter at the present epoch, was found to be $= 1.10 \pm 0.08$. They also found that the matter power spectrum,

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derived from the galaxy distribution P_{gg} data, differs from that derived from the CMB data by no more than 10% [16]. Using this result we examine the decay rate of the vacuum energy into CDM.

The paper is organized as follows. In Section II, we discuss the decay of the vacuum energy into CDM. Conclusions are presented in Section III.

II. VACUUM ENERGY DECAYING INTO CDM

A decaying vacuum energy into CDM increases the density of matter ρ, diluting the $(\delta\rho/\rho)$ spectrum. Consequently, a larger density fluctuation spectrum $(\delta\rho/\rho)^2$ is predicted at the recombination era ($z_{\rm rec} = 1070$) by the factor

$$F \equiv \left[\frac{\overline{\rho}_M(z)}{\overline{\rho}_M(z) - \Delta\rho(z)}\right]^2\Big|_{z=z_{\rm rec}}, \tag{1}$$

where

$$\overline{\rho}_M(z) = \rho_c^0(1+z)^3\Omega_M^0 \tag{2}$$

is the matter density for a constant vacuum energy density, $\rho_c^0 \equiv 3H_0^2/(8\pi G) \simeq 1.88h_0^2 \times 10^{-29} g\ \mathrm{cm}^{-3}$ is the critical density, and Ω_M^0 is the normalized matter density, $\Omega_M^0 = \rho_M^0/\rho_c^0\ (\sim 0.3)$.

The difference between the matter density $\bar{\rho}_M$ and the matter density ρ_{Mv} predicted by the model in which the vacuum energy decays into matter is

$$\Delta\rho(z) = \overline{\rho}_M(z) - \rho_{Mv}(z). \tag{3}$$

The density $\rho_{Mv}(z)$ is normalized at redshift $z = 0$ $[\rho_{Mv}(z=0) \equiv \rho_M^0]$. In order to describe the transfer of the vacuum energy ρ_Λ into matter ρ_{Mv} [1], we use the conservation of energy equation

$$\dot{\rho}_\Lambda + \dot{\rho}_{Mv} + 3H(\rho_{Mv} + P_{Mv}) = 0, \tag{4}$$

where P_{Mv} is the pressure due to ρ_{Mv}. For CDM, we have $P_{Mv} = 0$.

There exists an extensive list of phenomenological Λ-decay laws. Several models in the literature [17] are described by a power law dependence

$$\rho_\Lambda(z) = \rho_\Lambda^0(1+z)^n, \tag{5}$$

where $\rho_\Lambda^0 \equiv \rho_\Lambda(z=0)$, which we investigate here. Chen and Wu [4], for example, argued that $n = -2$ from dimensional considerations and general assumptions in line with quantum cosmology. In particular, they noted that this time variation of ρ_Λ leads to the creation of matter with a present rate which is comparable to that in the steady-state cosmology.

Following Peebles and Ratra [10], the solution for the matter density has the form

$$\rho_{Mv}(z) = A(1+z)^3 + B\rho_\Lambda(z), \tag{6}$$

where A and B are unknown constants. Using Eqs. (6) and (5) in Eq. (4), the dependence of ρ_{Mv} as a function of n in Eq. (5) is

$$\rho_{Mv}(z) = \rho_{Mv}^0(1+z)^3 - \left(\frac{n}{3-n}\right)\rho_\Lambda^0[(1+z)^3 - (1+z)^n]. \tag{7}$$

Using Eqs. (2) and (7) in Eq. (3), we find from Eq. (1) that

$$F = \left\{1 - \left(\frac{n}{3-n}\right)\left(\frac{\rho_\Lambda^0}{\rho_{Mv}^0}\right)[1-(1+z)^{n-3}]\right\}^{-2}. \tag{8}$$

If, as discussed in Section I, the density power spectrum from observations can be increased by no more than 10% due to the decay of the vacuum energy, we then have a maximum value for the F factor $F_{\rm max} = 1.1$. This maximum value gives $n_{\rm max} \approx 0.06$.

It is interesting to compare the vacuum energy density in the primordial nucleosynthesis era $\rho_{\Lambda PN}$, with the above value of $n_{\rm max}$ for the vacuum energy decay dependence given by Eq. (5). In the nucleosynthesis era ($z \sim 10^{10}$), we find that $\rho_{\Lambda PN} = \rho_\Lambda^0 10^{10n} = \rho_\Lambda^0 10^{0.6}$. Using $\rho_\Lambda^0 \cong 6h_0^2 \times 10^{-47}\ \mathrm{GeV}^4$ [14,15], we obtain $\rho_{\Lambda PN} \simeq 2h_0^2 \times 10^{-46}\ \mathrm{GeV}^4$. This is many orders of magnitude smaller than the maximum value $\rho_{\Lambda PN} \simeq 4.5 \times 10^{-12}\ \mathrm{GeV}^4$, obtained by Birkel and Sarkar [7] or $\rho_{\Lambda PN} = 1.1 \times 10^{-12}\ \mathrm{GeV}^4$, obtained by Freese *et al.* [6].

As noted above, Eq. (5) describes a power law dependence of ρ_Λ on the cosmic scale factor $a = (1+z)^{-1}$. Recently, Shapiro and Solà [12] suggested a first order time derivative dependence of ρ_Λ on a: $\rho_\Lambda \propto (da/dt)^2/a^2 \equiv H^2$, where H is the Hubble parameter. They were motivated by the renormalization group equation that may emerge from a quantum field theory formulation. They find a redshift dependence of the cosmological constants

$$\Lambda(z;\nu) = \Lambda_0 + \rho_c^0 f(z,\nu), \tag{9}$$

where $\Lambda(z=0) = \Lambda_0$, $k = 0$, and

$$f(z) = \frac{\nu}{1-\nu}[(1+z)^{3(1-\nu)} - 1]. \tag{10}$$

The dimensionless parameter ν in Eq. (10) comes from the renormalization group

$$\nu \equiv \frac{\sigma}{12\pi}\frac{M^2}{M_P^2}, \tag{11}$$

where σM^2 is the sum of all existing particles (fermions with $\sigma = -1$ and bosons with $\sigma = +1$). The range of ν is $\nu \in (0, 1)$ [18].

We take Eqs. (9) and (10) as a generic form for studying the decaying vacuum energy into CDM depending on a single parameter ν, regardless of its theoretical origin.

Using Eqs. (9) and (10), the matter density can be obtained as a function of z and ν in the matter era ($P_{Mv} = 0$):

$$\rho_{Mv}(z;\nu) = \rho_{Mv}^0(1+z)^{3(1-\nu)}. \tag{12}$$

Using Eqs. (12) and (2) in Eq. (3), the matter density difference $\Delta\rho$ at the recombination era is

$$\Delta\rho = \rho^0_{M\nu}(1 + z_{\rm rec})^3[(1 + z_{\rm rec})^{-3\nu} - 1]. \qquad (13)$$

The factor F modifying the density power spectrum is obtained, substituting Eqs. (2) and (13) in Eq. (1):

$$F = (1 + z_{\rm rec})^{6\nu}. \qquad (14)$$

Using $z_{\rm rec} \approx 1070$ and the maximum value of ν allowed in [12], $\nu = 0.1$, we find that $F \approx 66$. For the canonical choice $M^2 = M_P^2$ in Eq. (11), $\nu \approx 2.6 \times 10^{-2}$ and we obtain $F \approx 3$.

As noted above, observational data indicate that $F_{\rm max} = 1.1$. From this, we predict that $\nu_{\rm max}$ has a very small value, $\nu_{\rm max} \approx 2.3 \times 10^{-3}$.

III. CONCLUSIONS

We showed how the observed CMB and large galaxy survey data limit the vacuum energy decay rate into CDM between the recombination era and the present. When the vacuum energy decays into CDM, $\delta\rho/\rho$ is diluted. The density fluctuation spectrum is amplified by a factor F at the recombination era. From observations, the density power spectrum can be amplified by no more than 10% and the maximum value for F is $F_{\rm max} = 1.1$.

We investigate two forms for the decay of the vacuum energy ρ_Λ:

1 A general dependence on the cosmic scale factor a: $\rho_\Lambda(z, n) \propto a^{-n}$; and

2 A quadratic first derivative time dependence on the cosmic scale factor a: $\rho_\Lambda(z, \nu) \propto (da/dt)^2/a^2 \equiv H^2$, where $\rho_\Lambda = \text{const}$ for the parameter $\nu = 0$. We place upper limits on the values of n and ν.

We find that the decay of the vacuum energy into CDM as a scale factor power law $\rho_\Lambda \propto (1 + z)^n$, gives a maximum value for the exponent $n_{\rm max} \approx 0.06$. Similarly, for a parametrized vacuum decay into CDM model with $\Lambda(z; \nu) = \Lambda_0 + \rho^0_c[\nu/(1 - \nu)][(1 + z)^{3(1-\nu)} - 1]$, where ρ^0_c is the present critical density, we have an upper limit on the ν parameter, $\nu_{\rm max} = 2.3 \times 10^{-3}$.

Extrapolating ρ_Λ back to the primordial nucleosynthesis era with a dependence $\rho_{\Lambda PN} \propto (1 + z)^n$, we examine the predicted value for the vacuum energy density $\rho_{\Lambda PN}$ for the maximum value $n_{\rm max} = 0.06$. We obtain a maximum value for the vacuum energy $\rho_{\Lambda PN} \simeq 2h_0^2 \times 10^{-42}$ GeV^4. This can be compared with the Freese *et al.* [6] maximum value, $\rho_{\Lambda PN} = 1.1 \times 10^{-12}$ GeV^4, and the Birkel and Sarkar [7] maximum value, $\rho_{\Lambda PN} \simeq 4.5 \times 10^{-12}$ GeV^4. Thus, at the primordial nucleosynthesis era, we find for the above vacuum energy decay dependence, an upper limit for the vacuum energy density is 34 orders of magnitude smaller than in previous studies.

Because of the small values of $n_{\rm max}$ and ν_{max} our results indicate that if the vacuum energy is decaying into CDM, the rate of decay is extremely small.

ACKNOWLEDGMENTS

R. O. thanks the Brazilian agencies FAPESP (Grant No. 00/06770-2) and CNPq (Grant No. 300414/82-0) for partial support. A. M. P thanks FAPESP for financial support (Grant No. 03/04516-0 and No. 00/06770-2).

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